Enrollment No: ______________________ **Exam Seat No:**______________________

C. U. SHAH UNIVERSITY

Winter Examination-2020

Subject Name: Differential Equations

Subject Code: 5SC01DIE1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 09/03/2021** **Time: 11:00 To 02:00** **Marks: 70**

<u>Instructions:</u>

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1		**Attempt the Following questions**	**(07)**
	a.	Write The Bessel's differential equation.	**(01)**
	b.	What is the degree of the differential equation $\boldsymbol{x^4y'' + (y' - x)^{1/2} = 0}$?	**(01)**
	c.	Find the radius of convergence of $\sum_{\boldsymbol{n=1}}^{\infty} \boldsymbol{\frac{n!}{n^n} x^n}$.	**(01)**
	d.	Write the polynomial $(\boldsymbol{3x^2 - x - 1})$ in terms of Legendre polynomials.	**(02)**
	e.	Solve :$\frac{dy}{dx} = \frac{x-y}{x}$.	**(02)**
Q-2		**Attempt all questions**	**(14)**
	a.	Find the series solution of $(x-1)y'' + xy' + y = 0$ with $y(0) = 2$, $y'(0) = -1$.	**(07)**
	b.	Find the series solution of $8x^2y'' + 10xy' - (1+x)y = 0$near $x = 0$.	**(07)**
		OR	
Q-2		**Attempt all questions**	**(14)**
	a.	Find the series solution of $x^2y'' + x(x-1)y' + (1-x)y = 0$ near$x = 0$.	**(07)**
	b.	State and prove Orthogonality of Legendre's polynomials.	**(07)**
Q-3		**Attempt all questions**	**(14)**
	a.	State and Prove Rodrigue's Formula.	**(06)**
	b.	In usual notation prove that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)d\theta$ if $m = n$.	**(06)**
	c.	State Orthogonality of Bessel's function.	**(02)**
		OR	
Q-3		**Attempt all questions**	**(14)**
	a.	Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$ for all $n \geq 2$	**(06)**

	b.	Classify singularities: $x^3(x-1)y'' + 2(x-1)y' + 5xy = 0$	**(05)**
	c.	Prove that $\boldsymbol{J_n(x)}$ and $\boldsymbol{J_{-n}(x)}$ are linearly dependent**.**	**(03)**

SECTION – II

Q-4		**Attempt the following questions.**	**(07)**
	a.	Find $x\boldsymbol{F}(\boldsymbol{1},\boldsymbol{1},\boldsymbol{2},\boldsymbol{x})$.	**(02)**
	b.	The differential equation obtained from $\boldsymbol{z} = (\boldsymbol{x}-\boldsymbol{a})^{\boldsymbol{2}} + (\boldsymbol{y}-\boldsymbol{b})^{\boldsymbol{2}}$ by eliminating $\boldsymbol{a}$ and $\boldsymbol{b}$ is	**(02)**
	c.	State the condition when the given system of first order partial differential equation are compatible.	**(02)**
	d.	Solve: $z = px + qy + p^2 + q^2$.	**(01)**
Q-5		**Attempt all questions**	**(14)**
	a.	Solve $z^2 = pqxy$ using Charpit's method	**(05)**
	b.	Solve $y' = 2y - 2x^2 - 3, y(0) = 2$ using Picard's method of successive approximation up to 3 approximation.	**(05)**
	c.	Solve the partial differential equation ${u_x}^2 + {u_y}^2 + u_z$-1=0 by Jacobi's method.	**(04)**
		OR	
Q-5		**Attempt all questions**	**(14)**
	a.	Solve $p^2x + q^2y = z$ using Jacobi's method	**(05)**
	b.	Solve $(x^2 + y^2)p + 2xyq = (x+y)z$	**(05)**
	c.	Find a complete integral of$z^2p^2 + q^2 - p^2q$=0 using Charpit's method.	**(04)**
Q-6		**Attempt all questions**	**(14)**
	a.	Prove that: A necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$ a relation $F(u,v) = 0$, not involving x and y explicitly is that ,$\frac{\partial(u,v)}{\partial(x,y)} = 0$	**(08)**
	b.	Show that the Pfaffian differential equation $yzdx + 2xzdy - 3yxdz = 0$ is integrable and find its solution.	**(06)**
		OR	
Q-6		**Attempt all questions**	**(14)**
	a.	If$\vec{X} \cdot curl\ \vec{X} = 0$, where$\vec{X} = (P,Q,R)$ and μ is an arbitrary differential function of x, y and z then prove that $\mu\vec{X} \cdot curl\mu\vec{X} = 0$	**(05)**
	b.	Show that the Pfaffian differential equation $a^2y^2z^2dx + b^2x^2z^2dy + c^2x^2y^2dz = 0$ is integrable and find its solution.	**(05)**
	c.	Solve $xp + yq = z$	**(04)**